

Calculate the amount of work done in blowing the bubble of diameter 2 cm. Surface Tension of Soap Solution  $= 1.9 \times 10^{-2} \text{ N/m}$ .

$$\Rightarrow \text{Diameter} = \frac{0.02}{2} = 0.01 \text{ m}$$

Formula used.

$$W = T \cdot A$$

~~$$W = 1.9 \times 10^{-2} \times 0.01$$~~

~~$$W = 0.9$$~~

$$W = T \cdot A$$

$$\Rightarrow W = T \cdot 2\pi R^2$$

$$W = 1.9 \times 10^{-2} \times 8 \times 3.14 \times 1 \times 10^{-4}$$

$$W = 1.9 \times 2 \times 2.14 \times 10^{-6}$$

$$W = 1.9 \times 2.5 \times 10^{-6}$$

$$W = 4.75 \times 10^{-6}$$

$$\begin{array}{r} 0.004 \\ \times 0.001 \\ \hline 0.004 \\ 0.000 \\ \hline 0.004 \end{array}$$

$$\begin{array}{r} 1.9 \\ \times 2.5 \\ \hline 9.5 \\ 38 \\ \hline 4.75 \end{array}$$

The Radius of bubble of Soap Solution is 5 cm. How much work is needed to increase it to 7 cm. The Surface Tension of Soap Solution is  $0.03 \times 10^{-2} \text{ N/m}$ .

$$\Rightarrow W = T \Delta A$$

$$\Rightarrow \Delta A = 8\pi (R_2^2 - R_1^2)$$

$$\Delta A = 8\pi (0.0049 - 0.0025)$$

$$= 8\pi \times 0.0024$$

$$\Delta A \Rightarrow 0.0192 \pi$$

$$\begin{array}{r} 0.0049 \\ - 0.0025 \\ \hline 0.0024 \end{array}$$

$$W = 0.03 \times 0.0192 \times \pi$$

Q. A Drop of water of Diameter = 0.2 cm is split into 27000 Droplets of same size. Assuming the temperature to remain constant to work done. Surface tension of water =  $7 \times 10^{-2} \text{ N/m}$

$$W = 4\pi R^2 T \cdot \left( n^{\frac{1}{3}} - 1 \right)$$

$$R = 1 \times 10^{-3} \text{ m}$$

→

$$W = 4 \times \pi \times 1 \times 10^{-3} \times 1 \times 10^{-3} \times 7 \times 10^{-2} \left( 30^{\frac{1}{3}} - 1 \right)$$

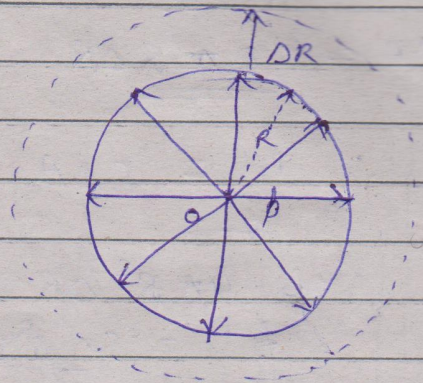
$$W = 4 \times 3.14 \times 1 \times 10^{-6} \times 7 \times 10^{-2} \times 29$$

$$W = 12.56 \times 10^{-6} \times 7 \times 10^{-2} \times 29$$

$$W = \cancel{2.55 \times 10^{-5}} \\ \underline{2.55 \times 10^{-5}}$$

Q) Derive the expression for axis pressure inside liquid drop.

Consider a liquid drop of radius  $R$ . The molecules lying on the surface of liquid drop due to surface tension will experience a resultant force acting inwards  $\perp$  to the surface. Since the size of liquid drop cannot be reduced zero due to force of surface tension therefore the pressure inside the drop must be greater than the pressure outside. Thus the exertion of pressure inside the drop will provide a force acting outwards  $\perp$  to the surface, counter balance the resultant force due to surface tension.



Let  $S$  surface tension of liquid drop  
 $p$  equal axis of press- inside the drop  
 $\Delta R =$  Increase in radius of the drop due to axis of pressure.

Then work done by the axis pressure

$$W = F \times \text{displacement}$$

$$W = F \times \Delta R$$

$$W = \text{displacement} \times \text{area} \times \text{increase in radius}$$

$$W = p \times 4\pi R^2 \times \Delta R \quad \text{--- (1)}$$

Increase in surface area of drop = final surface area - initial surface area.

$$\Delta A = 4\pi (R + \Delta R)^2 - 4\pi R^2$$

$$\Delta A = 4\pi [(R + \Delta R)^2 - R^2]$$

$$\Delta A = 4\pi [R^2 + (\Delta R)^2 + 2R\Delta R - R^2]$$

$$\Delta A = 4\pi [(\Delta R)^2 + 2R\Delta R]$$

$$\Delta A = 4\pi \times 2R \Delta R \quad \left[ \because \Delta R^2 \text{ is a much smaller quantity so it can be neglected.} \right]$$

$$\Delta A = \underline{8\pi R \Delta R}$$

$$\therefore W = S \Delta A$$

$$W =$$

$$p \cdot 4\pi R^2 \Delta R = 8\pi R \Delta R \cdot S$$

$$p = \frac{S \cdot 8\pi R \Delta R}{4\pi R^2 \Delta R}$$

$$p = \frac{2S}{R}$$

Ques Derive the expression for excess pressure inside a Soap bubble.

$$W = p \times 4\pi R^2 \times \Delta R$$

$$\Delta A = 2 \times 4\pi [R + \Delta R]^2 - 4\pi R^2$$

$$\Delta A = 2 \times 4\pi [(R + \Delta R)^2 - R^2]$$

$$\Delta A = 2 \times 4\pi [R^2 + (\Delta R)^2 + 2R\Delta R - R^2]$$

$$\Delta A = 2 \times 4\pi [( \Delta R)^2 + 2R\Delta R]$$

$$\Delta A = 2 \times 4\pi \times 2R\Delta R$$

$$\Delta A = 16\pi R \Delta R \quad \text{(2)}$$

$$\therefore W = S \Delta A$$

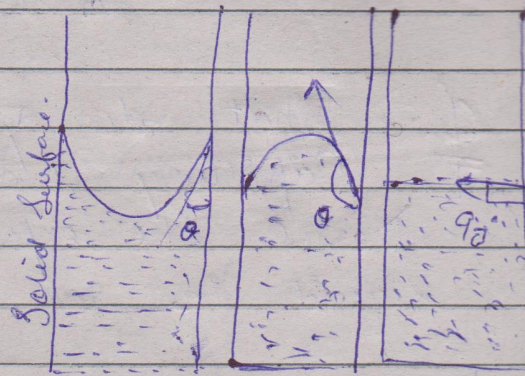
$$p \cdot 4\pi R^2 \Delta R = S \cdot 16\pi R \Delta R$$

$$p = \frac{S \cdot 16\pi R \Delta R}{4\pi R^2 \Delta R}$$

$$p = \frac{4S}{R}$$

Q. What do you mean by angle of contact. Write the effects of angle of contact.

The angle inside the liquid which the tangent to the liquid surface at the point of contact makes with the solid surface is called the angle of contact for that pair of solid and liquid.



### EFFECTS OF ANGLE OF CONTACT

The liquid sticks the solid surface if the angle of contact for a pair of liquid and solid is  $90^\circ$  such as water.

Water sticks with the glass. The liquid

does not stick with the solid surface if

the angle of contact for that pair of liquid and solid  $> 90^\circ$  such as

Mercury does not stick with the glass.

⇒ If angle of contact is less than  $90^\circ$  the meniscus of liquid in a capillary tube is concave and the liquid rises in the capillary.

⇒ If the angle of contact is greater than  $90^\circ$  the meniscus of liquid in the capillary tube is convex and the liquid falls down in the capillary.

or fall down. Rises in Capillary  
 Surface of liquid in the free  
 is horizontal. Capillary

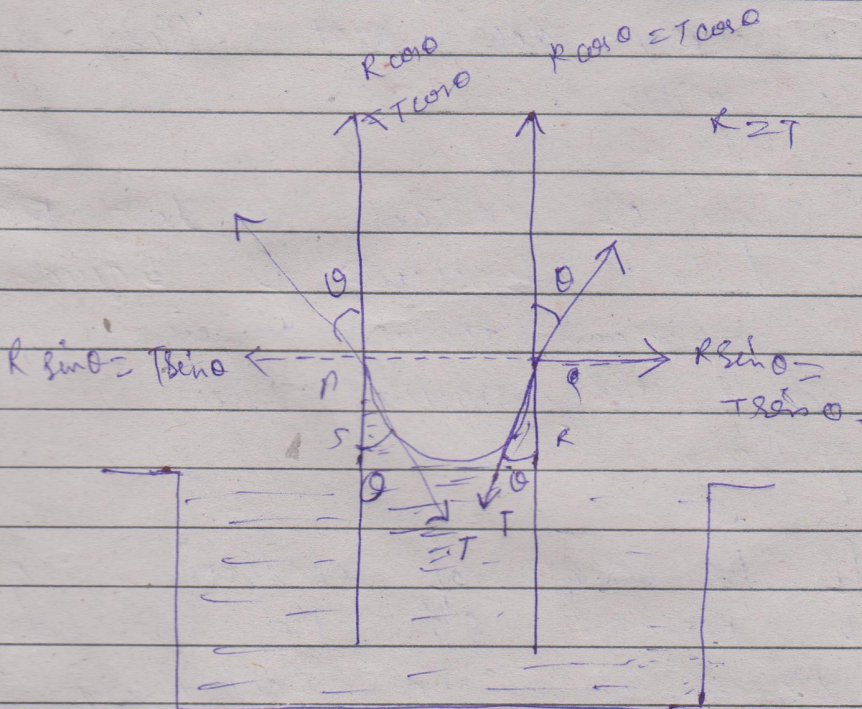
Q. What do you mean by  
 Capillary and Capillarity.

Derive the expression for rise of  
 liquid in a capillary tube. (Hagen  
 formula)

\* Capillary :- A tube with a fine and  
 uniform bore throughout  
 its length is called a  
 capillary tube.

\* Capillarity :- The phenomenon of rise  
 of fall down in liquid  
 in a capillary tube is  
 called capillarity.

\* Expression for rise of liquid in a  
 capillary tube :-



$h$  = height to which the liquid rises and shown in diag.

$\rho$  = Density of liquid at room Temp

$\theta$  = Angle of contact

$T$  = Force of Surface Tension acting Tangentially Downward to liquid Meniscus.

$R$  = Reaction force acting Tangentially upward to the liquid Meniscus

but  $R = T$

The Reaction force  $R = T$  can be resolve into 2 Rectangular components

$R \cos \theta = T \cos \theta$  Acting upward

$R \sin \theta = T \sin \theta$  Acting  $\perp$  to the Wall of the tube.

This component therefore play no part in rising the liquid in the tube.

Only the component responsible for the rising the liquid in the tube is  $T \cos \theta$ .

The  $T \cos \theta$  acts all along the circumference of the circle of contact of the liquid with the tube hence,

total upward force

~~$F = T \cos \theta$~~  (D)

$F = T \cos \theta \cdot 2\pi r$  (D)

The vol. of liquid in the cylindrical column of height  $h$  and radius  $r$  is  $\pi r^2 h$ .

The vol. of liquid under the meniscus  
 equal to the vol. of cylinder PQRS of  
 height  $r$  and Radius  $R$  —  
 volume of hemisphere.

$$\Rightarrow \pi r^2 \cdot r - \frac{2}{3} \pi r^3$$

$$\Rightarrow \pi r^3 - \frac{2}{3} \pi r^3$$

$$= \frac{3\pi r^3 - 2\pi r^3}{3}$$

$$= \frac{\pi r^3}{3}$$

Total vol. of liquid in the  
 capillary tube =

$$V = \pi r^2 h + \frac{\pi r^3}{3}$$

$$V = \pi r^2 \left( h + \frac{r}{3} \right)$$

Weight of Raised liquid in the tube

$$W = mg$$

$$W = V \rho g$$

$$W = \pi r^2 \left( h + \frac{r}{3} \right) \rho g \quad \text{--- (2)}$$

In equilibrium  
 $F = W$

$$\text{--- (1)} = T \cos \theta \cdot 2\pi r = \pi r^2 \left( h + \frac{r}{3} \right) \rho g$$

$$2\pi r \cos \theta$$

$$T = \frac{r \left( h + \frac{r}{3} \right) \rho \cdot g}{2 \cos \theta}$$

But  $h \gg r/3$

$\Rightarrow$  so,  $r/3$  can be neglected.

$$\Rightarrow \therefore T = \frac{r h \rho g}{2 \cos \theta} \quad \text{--- (3)}$$

$\Rightarrow$  For the pure water and clean glass  $\theta \approx 0$   
 $\cos \theta = 1$

$$T = \frac{r h \rho g}{2}$$

$$\{ r_1 h_1 = r_2 h_2 \}$$

Q What are the factors affecting the Surface Tension.

Following factors affect the Surface Tension of a Liquid:-

$\Rightarrow$  effect of Temperature:-

The Surface Tension of a liquid increases with the rise in temperature

The Surface Tension of liquid at temp  $t^{\circ}\text{C}$ .

$$T_t = T_0 (1 - \alpha t)$$

$T_t, T_0$  are the Surface Tension at  $T^{\circ}\text{C}$  and  $0^{\circ}\text{C}$  respectively and  $\alpha$  is the Temp. coefficient of Surface Tension.

\* effect of Soluble impurities:-

Due to the presence of Soluble impurities in a liquid the Surface Tension either decreases or increases.

If the impurities is less Soluble, the Surface Tension of liquid is Increase.

EX-1 On Adding soap or ~~fat~~ in water the Surface Tension of water decreases. <sup>But</sup> If the impurities is More Soluble the Surface Tension of liquid Increase.

On dissolving Salt in water the Surface Tension of Salt Solution is more than that of pure water.

\* effect of insoluble impurities:-

If such a substance is mixed in a liquid which does not dissolve in it but that contaminated on its surface the Surface Tension of a liquid Decreases.

Tension of water decreases.

Ques. The small drops of liquid are spherical while large ones are flat. Why?

→ The Reason is that when a drop of liquid is kept on a glass plate. The shape of the drop depends on the relative magnitude of forces.

1) Force of surface tension due to which surface area of liquid tends to become minimum.

2) The force of gravity due to which the centre of gravity <sup>must</sup> equal to acquire the lowest position.

For a small drop the force of gravity is negligible in comparison to the force of surface tension. Hence drop takes the spherical shape.

For a big drop the force of gravity is not negligible. Due to the force of gravity the centre of gravity of the drop tends to acquire the lowest position due to which the drop becomes flat.

Determine the radius of new bubble form when two bubbles coalesce together.

Consider two bubbles of Radii  $r_1$  and  $r_2$  respectively. If  $V_1$  and  $V_2$  are the volume of two bubble then

$$V_1 = \frac{4}{3} \pi r_1^3 \quad \text{and}$$

$$V_2 = \frac{4}{3} \pi r_2^3$$

Let  $T$  be the surface Tension of the solution if  $P_1$  and  $P_2$  are the excess pressure inside the two bubble

$$P_1 = \frac{4T}{r_1} \quad \text{and} \quad P_2 = \frac{4T}{r_2}$$

Let  $r$  be the Radius of New bubble  
 $V$  be the volume of New bubble  
 $P$  be the excess of pressure inside the New bubble.

$$V = \frac{4}{3} \pi r^3 \quad \text{and} \quad P = \frac{4T}{r}$$

Assuming the external pressure negligible

And new bubble is formed under isothermal condition by Boyle's law

$$\Rightarrow PV = P_1 V_1 + P_2 V_2$$

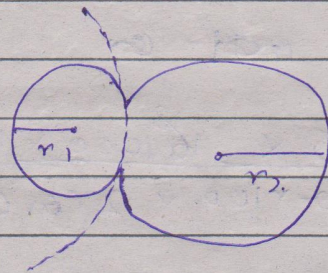
$$\frac{4T}{r} \times \frac{4}{3} \pi r^3 = \frac{4T}{r_1} \times \frac{4}{3} \pi r_1^3 + \frac{4T}{r_2} \times \frac{4}{3} \pi r_2^3$$

$$r^2 = r_1^2 + r_2^2$$

$$r = \sqrt{r_1^2 + r_2^2}$$

Ques. Two soap bubbles of radii  $r_1$  and  $r_2$  collapse together to form a double bubble. Calculate the radius of curvature of its inner common interface.

$\Rightarrow$  Consider two soap bubbles of radii  $r_1$  and  $r_2$  in contact with each other



as shown in fig.

Let  $r$  be the radius of the common boundary. If  $P_1$  and  $P_2$  are the excess pressures of the two sides of the interface. Then resultant pressure.

$$P = P_1 - P_2$$
$$\frac{4T}{r} = \frac{4T}{r_1} - \frac{4T}{r_2}$$

$$\frac{1}{r} = \frac{1}{r_1} - \frac{1}{r_2}$$

$$\frac{1}{r} = \frac{r_2 - r_1}{r_1 r_2}$$

$$r = \frac{r_1 r_2}{r_2 - r_1}$$